

**CONTROL AND SCHEDULING FOR SYSTEMS AFFECTED BY  
RANDOM PACKET LOSS**

**Edwin G.W. Peters (MSc AAU, BSc AAU)**

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School of Electrical Engineering and Computing

Supervision by: Prof. Minyue Fu, Dr. Damián Marelli and Prof. Daniel Quevedo

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## Statement of originality

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I hereby certify that the work embodied in the thesis is my own work, conducted under normal supervision.

The thesis contains no material which has been accepted, or is being examined, for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to the final version of my thesis being made available worldwide when deposited in the University's Digital Repository, subject to the provisions of the Copyright Act 1968 and any approved embargo.

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# Abstract

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Modern technologies in manufacturing, transportation, energy generation and distribution, smart buildings, smart cities and so on heavily involve large-scale, distributed networked control systems (NCSs). Nowadays, the communication between sub-systems within a NCS occurs mostly over wired networks. However, in large scale systems, the cost of installation and maintenance of these wired networks can be significant.

One method of cost reduction while also gaining flexibility, is to replace these wired networks with wireless networks. Utilizing wireless networks within NCSs allows for greater flexibility, since no new cables have to be installed when adding new sensors and/or actuators. Further, it opens up for the use of remote sensors and actuators in locations, that are prohibitive with cabled networks. These sensors and actuators can be battery powered and mounted on moving objects, such as vehicles or drones. This opens up for entirely new possibilities for control and sensing within NCSs.

However, wireless networks introduce network effects that can severely affect the performance of the sub-systems, and in some cases, lead to instability. These effects include, but are not limited to, congestion, interference, packet loss and bandwidth limitations. In this thesis, we address the controller and estimator design for NCSs that are connected with wireless networks. We show, that designs that take these network effects into account can not only achieve increased performance, but also guaranteed closed-loop stability.

The first part of this thesis considers the synthesis and analysis of controllers and estimators for networks affected by random packet loss. In particular, Chapter 3 considers controller and estimator synthesis and analysis for linear systems affected by independent and identically distributed packet loss. We establish a form of duality that extends the duality in the classical linear quadratic Gaussian result to the design for systems affected by packet loss. Chapter 4 extends the results from Chapter 3 and presents a method to synthesise controllers for networks where the packet loss model contains memory as well. This however results in a large number of controllers. To reduce the amount of controllers, we present three methods that trade-off controller complexity and control performance.

The second part of this thesis (Chapters 5 and 6) considers the control design problem for a large distributed system with a bandwidth limited wireless network. The wireless transmission protocol features a limited number of reliable transmission slots with negligible packet dropouts and a more widely available transmission period, where packet collisions and delays occur more

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frequently. We propose a controller and scheduler co-design that optimally selects both the schedule on which actuators to address, and the control inputs for the sub-systems that are addressed. Simulation studies illustrate that the online optimal co-design method results in significantly improved performance over heuristic scheduling. However, the computational complexity for the online algorithms makes practical implementations of the proposed method prohibitive. To reduce the computational complexity, the design is extended by the use of a novel model predictive control (MPC) algorithm that combines approximations to infinite horizon cost functions with a short online prediction horizon. This results in improved control performance while maintaining relatively low computational complexity.

In this thesis, we show that taking networks effects, such as packet loss and bandwidth limitations, directly into account in the controller and estimator design phase leads to significant performance gains and in some cases, guaranteed closed-loop stability.

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# Notation

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$\triangleq$	Definition
$(\dots)$	A sequence
$\{\dots\}$	A set
$\mathbb{R}$	The set of real numbers
$\mathbb{R}^n$	Vector with $n$ elements taking values in the set $\mathbb{R}$
$\mathbb{R}^{n \times m}$	Matrix with $n$ rows and $m$ columns taking values in the set $\mathbb{R}$
$\mathbb{N}$	The set of natural numbers
$\mathbb{Z}$	The set of all integers
$\mathbb{Z}_a$	The set of integers $\{0, 1, \dots, a - 1\}$
$\text{card}(\mathcal{A})$	The cardinality of the set $\mathcal{A}$
$A > 0$ ( $A < 0$ )	The matrix $A$ is positive (negative) definite
$A \geq 0$ ( $A \leq 0$ )	The matrix $A$ is positive (negative) semi-definite
$A \geq B$	The matrix $A - B \geq 0$
$A^T$	Transpose of the matrix $A$
$A^{-1}$	The inverse of matrix $A$
$A^\dagger$	Pseudo inverse of matrix $A$
$\text{trace}(A)$	Trace of the square matrix $A$
$\sigma(A)$	Spectral radius of the matrix $A$
$\text{diag}\{X\}$	The matrix with $X$ on its (block-)diagonal
$I_n$	The $n \times n$ matrix with ones on its diagonal
$\mathbf{0}_{n \times m}$	The $n \times m$ matrix containing zeros
$a_i$	The $i$ 'th column vector in the matrix $A$
$a_{ji}$	The $j$ 'th element in the $i$ 'th column vector of the matrix $A$
$\ x\ $	Two-norm of $x$
$\ x\ _Q^2$	Weighted norm of $x$ squared i.e. $x^T Q x$
$\lceil x \rceil$	Rounds $x$ up to the nearest larger integer
$\lfloor x \rfloor$	Rounds $x$ down to the nearest lower integer
$x \% N$	$x$ modulo $N$ returns the remainder after division for $\frac{x}{N}$
$\Pr\{X\}$	The probability for the event $X$
$\Pr\{X Y\}$	The probability for the event $X$ knowing $Y$
$\Pr\{X; Z\}$	The probability for the event $X$ which distribution is parameterized by $Z$

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$\mathbf{E}\{X\}$	Expected value of the random variable $X$
$\mathbf{E}\{X Y\}$	Expected value of the random variable $X$ knowing $Y$
$\mathbf{E}\{X; Z\}$	Expected value of the random variable $X$ which distribution is parameterized by $Z$
$\mathbb{1}_{\mathcal{S}}(y)$	The indicator function returns one if $y \in \mathcal{S}$ or zero else
$\mathcal{U}(i, j)$	Uniform distribution in the interval $[i, j]$
$\mathcal{N}(\mu, \Sigma)$	Gaussian distribution with mean $\mu$ and (co-)variance $\Sigma$
$x \sim \mathcal{N}(\mu, \Sigma)$	The random variable $x$ is Gaussian distributed
$x \sim \mathcal{U}(i, j)$	The random variable $x$ is uniformly distributed
$\otimes$	The Kronecker product
$\text{vec}(A)$	The vector of the columns of $A$ stacked i.e. $\begin{bmatrix} a_1^T & a_2^T & \cdots & a_n^T \end{bmatrix}^T$
$\circ$	The composition of functions
$n!$	The factorial $n! = n(n-1)(n-2) \cdots 1$

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## Abbreviations

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BI	beacon interval
CA	cost averaged
CAP	contention access period
CFP	contention free period
CSMA	carrier sense multiple access
CSMA/CA	carrier sense multiple access with collision avoidance
FDMA	frequency division multiple access
flop	floating point operation
FSMC	finite state Markov chain
GA	group averaged
GTS	guaranteed time-slot
<i>i.i.d</i>	independent and identically distributed
JLS	jump linear system
LQ	linear quadratic
LQG	linear quadratic Gaussian
LQR	linear quadratic regulator
LTl	linear time-invariant
MA	Markov averaged
MJLS	Markov jump linear system
MMSE	minimum mean square error
MPC	model predictive control
MSS	mean square stability
NCS	networked control system
NP	non-deterministic polynomial-time
PAN	personal area network
RR	round robin
SDP	semi-definite program
TCP	transmission control protocol
TDMA	time division multiple access
UDP	user datagram protocol



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